7 More about Equations

Mock Questions

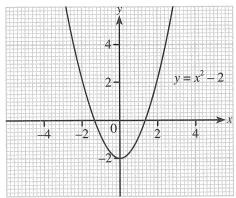
In the following questions, unless otherwise specified, numerical answers must be either exact or correct to 3 significant figures.

Section B

- 1. Solve $2x 1 \frac{2}{x + 1} = 0$. (3 marks)
- 2. Solve $\left(1 + \frac{1}{x}\right)(x+8) = 3$. (3 marks)
- 3. Solve $\frac{x}{x+1} \frac{x^2 6}{x+1} = 3$. (3 marks)
- 4. Solve $\frac{3x}{x-2} \frac{8}{2x-3} = 5$. (3 marks)
- 5. Solve $u^4 25u^2 + 144 = 0$. (4 marks)
- **6.** Solve $216x^6 37x^3 8 = 0$. (4 marks)
- 7. Solve $x^2(2x^2 + 9) = 35$ and give the answers in surd form. (4 marks)
- 8. Solve $\sqrt{2x+13}+1=x$. (4 marks)
- 9. Solve $3x^3 191x\sqrt{x} 64 = 0$. (4 marks)
- **10.** Solve $\sqrt{x^2 + 5x + 10} = 4$. (3 marks)
- 11. Solve $5(2x+1)^{\frac{2}{3}} (2x+1)^{\frac{4}{3}} 4 = 0.$ (4 marks)
- 12. Solve $5^{2x} 24(5^x) 25 = 0$. (4 marks)
- 13. Solve $3^{x}(3^{x+1} + 14) 5 = 0$. (4 marks)
- **14.** Solve $9^x + 5(3^x) = 36$. (4 marks)

- 15. Solve $2^x + 3 2^{2-x} = 0$. (4 marks)
- **16.** Solve $(\log x)^2 \log x^3 + 2 = 0$. (4 marks)
- 17. Solve $\log (x-4) + \log (x+5) = 1$. (4 marks)
- 18. Solve $\log_3 \frac{1}{\sqrt{2x+1}} = \log_3 \sqrt{x-3} 1$. (4 marks)
- 19. Solve $2\cos^2\theta + 5\cos\theta + 2 = 0$, where $0^{\circ} \le \theta < 360^{\circ}$. (3 marks)
- **20.** Solve $2 \sin^2 \theta + 7 \cos \theta 7 = 0$, where $0^{\circ} \le \theta < 360^{\circ}$. (4 marks)
- 21. Solve $\tan^2 \theta + 4 \tan \theta 5 = 0$, where $0^\circ \le \theta < 360^\circ$. (4 marks)
- 22. Solve $\sin^2 \theta 3 \sin \theta \cos \theta + 2 \cos^2 \theta = 0$, where $0^\circ \le \theta < 360^\circ$. (5 marks)
- 23. Solve $\frac{3}{x-2} \frac{1}{x} = \frac{1}{2(x+2)}$. (5 marks)
- **24.** (a) Solve $u^2 + 2u 8 = 0$. (1 mark)
 - (b) Hence solve the equation $\left(\frac{1}{1-t^2}\right)^2 + \frac{2}{1-t^2} 8 = 0.$ (4 marks) (Give the answers in surd form if necessary.)
- **25.** (a) Solve $2u^2 7u 15 = 0$. (1 mark)
 - (b) Hence solve the equation $2(x^2 4x) 7(x^2 4x) 15 = 0$. (Give the answers in surd form if necessary.) (4 marks)
- **26.** (a) Solve $\frac{6}{x+2} = 5 x$. (2 marks)
 - (b) Hence solve the equation $\frac{6}{x^2 + 3x + 2} + x^2 + 3x 5 = 0$. (Give the answers in surd form if necessary.) (4 marks)
- 27. (a) Solve $3x^2 + 14x 24 = 0$. (1 mark)
 - (b) By using part (a), solve $3x^7 + 14x^4 = 24x$. (4 marks)
- 28. Solve the equation $\left(x + \frac{2}{x}\right)^2 3\left(x + \frac{2}{x}\right) 18 = 0$. (Give the answers in surd form if necessary.) (5 marks)

29. The figure shows the graph of $y = x^2 - 2$.



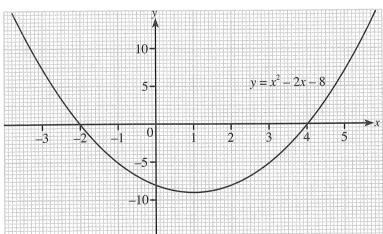
(a) By drawing a suitable line on the graph, solve each of the following simultaneous equations. (Give the answers correct to 1 decimal place.)

(i)
$$\begin{cases} y = x^2 - 2 \\ x - 2y + 5 = 0 \end{cases}$$

(ii)
$$\begin{cases} y = x^2 - 2 \\ x + y - 3 = 0 \end{cases}$$

(4 mark

- **(b)** Elaine suggests that she can use the graph of $y = x^2 2$ to solve $x^2 4x 5 = 0$. Do you agree? Explain (1 marl your answer.
- **30.** The figure shows the graph of $y = x^2 2x 8$.



(a) Using the graph, solve each of the following equations. (Give the answers correct to 1 decimal place.)

(i)
$$x^2 - 2x - 11 = 0$$

(ii)
$$x^2 - 4x - 4 = 0$$

(6 mark

(5 mark

- (b) Suggest an equation of straight line such that the line meets the graph of $y = x^2 2x 8$ at only one (1 mar point.
- 31. Solve the simultaneous equations $\begin{cases} y = (x+1)^2 \\ 2x + y + 3 = 0 \end{cases}$

32. Solve $\begin{cases} y = x^2 - 4x - 5 \\ 2x + y - 10 = 0 \end{cases}$

(5 marks)

- 33. Solve the simultaneous equations $\begin{cases} y = (x-2)(x-1) + 1 \\ y = 3x 6 \end{cases}$ (5 marks)
- **34.** Solve $x^2 + y^2 24 = x y = 1$. (5 marks)
- 35. Solve the simultaneous equations $\begin{cases} x^2 2xy + y^2 = 0 \\ x 2y + 3 = 0 \end{cases}$ (5 marks)
- **36.** If $\begin{cases} y = x^2 2x + 1 \\ y = mx 3 \end{cases}$ have only one real solution, find the values of m. (4 marks)
- 37. If the graphs of y = 5x + 3 and $y = x^2 + x + k$ have two distinct points of intersection, find the range of values of k. (4 marks)
- 38. If $\begin{cases} y = 3x^2 + 4x + 7 k \\ 2x y + 1 = 0 \end{cases}$ have real solutions, find the range of values of k. (4 marks)
- 39. The graph of $y = x^2 + kx + 3$ intersects the straight line y = 4x + k at exactly one point.
 - (a) Find the value of k.

(3 marks)

(b) Find the point of intersection.

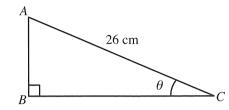
(3 marks)

- 40. If the sum of the reciprocals of two positive consecutive even integers is $\frac{13}{84}$, find the value of the smaller (4 marks)
- 41. (a) Kay wants to spend \$48 to buy apples in a supermarket. If the price of each apple is reduced by 80 cents, she can buy 3 more apples without any changes. Find the original price of each apple.

(4 marks)

- (b) The cost of each apple is \$3.5 and the apples are sold at the original price in (a). The manager of the supermarket claims that the supermarket can make at least 10% of profit by selling all the apples. Do you agree? Explain your answer. (2 marks)
- 42. A 2-digit positive number is decreased by 27 when the digits are interchanged. If the product of the 2 digits is 28, find the original number. (5 marks)

- **43.** Eric walked 300 m from *A* to *B* at an uniform speed. Then he reduced his speed by 20 m/min and walked 400 m from *B* to *C*. The total time taken for him to walk from *A* to *C* is 8 minutes.
 - (a) Find his final speed. (4 marks)
- Eric thinks that if he walks from A to C with the speed obtained in (a), the extra time he needs should not exceed 30 s. Do you agree? Explain your answer. (3 marks)
- **44.** The figure shows a right-angled triangle ABC with perimeter 60 cm. AC = 26 cm and $\tan \theta < 1$.



(a) Find AB and BC.

(4 marks)

(b) If D is a point on AC such that BD is an altitude of $\triangle ABC$ on AC, find AD.

(3 marks)

- **45.** A cannonball flies at a horizontal speed of 100 m/s when it is fired. Two cars *A* and *B* were 420 m from two cannons *X* and *Y* respectively. Cannon *X* fired a cannonball to car *A* while cannon *Y* fired a cannonball to car *B* at the same time. At the same moment, car *A* moved towards cannon *X* at a speed of *x* m/s, while car *B* moved away from cannon *Y* at the speed of *x* m/s. Both cannonballs hit the cars.
 - (a) Express the times needed for the cannonballs to hit cars A and B in terms of x.
 - **(b)** If car *B* is hit 4 seconds after car *A* is hit, find *x*.

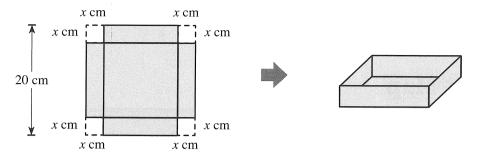
(4 marks) (3 marks)

- **46.** A company is developing a network for generating electricity. The network consists of some generators. When there are x generators in the network, each of them can generate $(x^3 15x)$ units of electricity per day. The electricity generated each day will be transmitted to the users with a total loss of 50 units.
 - (a) If y is the total amount of electricity that the users received per day, prove that $y = x^4 15x^2 50$.

(2 marks)

- (b) If the total amount of electricity transmitted to the users is 706 units, find the number of generators required. (4 marks)
- (c) The cost of running a generator per day is $$62x^3 1800x$. If the electricity is charged on the users at \$60 per unit, using the result in (a), find the number of generators and the corresponding profit such that the profit per day is maximum. (4 marks)

47. The figure below shows a square cardboard of side 20 cm. A small square of side x cm is cut from each corner of the cardboard. The remaining part is then folded into an open container as shown.



(a) Show that the capacity $V \text{ cm}^3$ of the box is given by $V = 4x^3 - 80x^2 + 400x$.

(1 mark)

- (b) (i) Factorize the polynomial $x^3 20x^2 + 100x 144$.
 - (ii) Find the height of the box if its capacity is 576 cm³. (Give the answers in surd form if necessary.)

(5 marks)

- (c) Annie wants to modify the container into a gift box. She adds a paper cube of side x cm onto each of the corner of the container and then place a square platform of dimensions $4 \text{ cm} \times 4 \text{ cm} \times 1 \text{ cm}$ at the centre of its base.
 - (i) If the capacity of the gift box is 304 cm³, find its height.
 - (ii) Can the capcity of the gift box be larger than 500 cm³? Explain your answer.

(5 marks)